Int. J. Solids Structures Vol. 29, No. 14/15, pp. 1711–1721, 1992
Printed in Great Britain.
0020–7683/92 $5.00 + .00
Pergamon Press plc

TRANSIENT RESPONSE OF A LAMINATED COMPOSITE PLATE: RESULTS FROM HOMOGENIZATION AND DISCRETIZATION

S. K. DATTA†, T.-H. JU and R. L. BRATTON
Department of Mechanical Engineering, CIRES, and Center for Space Construction, University of Colorado, Boulder, CO 80309-0427, U.S.A.

and

A. H. SHAH
Department of Civil Engineering, University of Manitoba, Winnipeg, Canada R3T 2N2

Abstract—Transient response of a multilayered laminated plate has been studied in this paper. The objective of this study is to analyze the effect of layering on the response of the laminated plate in both time and frequency domains to a line source on the surface of the plate. For simplicity of analysis, attention has been focused on the two-dimensional (plane strain) motion. It is shown that for a cross-ply plate when the number of plies is small the response of the plate is quite different than that of an equivalent homogeneous plate. For short pulses the response is fairly complicated due to the reflections from the interfaces between the plies. Dispersion of waves in the plate is also analyzed. It is found that the homogenized model predictions for the low-order modes agree with those of the layered model. However, they diverge when high-order modes are considered. This is consistent with the transient response comparisons.

INTRODUCTION

Propagation of guided waves in a laminated plate is of interest for ultrasonic nondestructive evaluation of defects and for material characterization. There is a need for a thorough understanding of the wave propagation characteristics in such a plate in order to use ultrasonic means to determine the material properties, assess damage, and characterize defects. The problem is also of interest for study of acoustic emission.

Dispersive behavior of guided waves in laminated plates has been studied extensively in recent years. It has been shown that dispersive modal propagation behavior is strongly influenced by the anisotropic properties of each ply and stacking sequence used. Thus this can be used to determine material properties. Propagation of free guided waves (Lamb waves) in an anisotropic homogeneous plate has been studied in detail recently by Nayfeh and Chimenti (1988b) and Li and Thompson (1990). These studies provide an interesting picture of the rich dispersion characteristics of the Lamb waves. They also contain a comprehensive survey of the literature on guided waves in homogeneous anisotropic plates. A comprehensive review of current work (theoretical and experimental) can also be found in the edited volumes of Mal and Ting (1988) and Datta *et al.* (1990). Among the theoretical works are those of Kaul and Mindlin (1962), Abubakar (1962), Solie and Auld (1973), Baylis and Green (1986), Mal (1988), Datta *et al.* (1988), Bratton *et al.* (1989, 1990), and Dayal and Kinra (1989).

In the studies mentioned above attention was focused on free Lamb waves. The effect of fluid loading on ultrasonic guided waves in composite plates has also been investigated by several authors. References to these can be found in the two edited volumes mentioned above and in the works by Nayfeh and Chimenti (1988a,b) and Mal and Bar-Cohen (1989).

Guided waves in layered (laminated) composite plates have also received attention. For references the reader is referred to the edited volumes mentioned above and to the papers by Dong and Nelson (1972), Dong and Hwang (1985), Mal (1988), Datta *et al.* (1988), Dayal and Kinra (1989), and Chimenti and Nayfeh (1990). A systematic investigation of the effect of increasing number of laminae on the dispersion of free guided waves in a laminated plate has been reported recently by Karunasena *et al.* (1991). Since in many

† Presently at the Department of Mechanical Engineering, University of Connecticut, Storrs, CT 06269-3139, U.S.A.

structural applications there are usually many laminae in a composite plate (shell) it is of interest to investigate the effect of the number of layers on the dispersion behavior.

As is evidenced from the above review, there is now a considerable body of literature on dispersive guided waves. However, very few studies have been reported dealing with the propagation of disturbances in a laminated plate generated by sources. Early works on wave propagation in an anisotropic plate due to impact are by Moon (1972), and Whitney and Sun (1973), who used Mindlin plate theory to study the dynamic response of a composite plate. Recently, Chang and Sun (1988) have used a finite difference procedure to analyze the two-dimensional (plane stress) problem of wave propagation in a homogeneous orthotropic plate. Pulse propagation in a fluid-loaded laminated plate when the source is in the fluid has also been studied by Mal *et al.* (in press).

In this paper we report on our investigation of the effect of layering on the transient wave propagation in a laminated cross-ply plate. For simplicity of analysis the attention is focused here on the two-dimensional (plane strain) problem when a line vertical force is applied on a free surface of the plate, the line being parallel or perpendicular to the fibers in a ply. Results in both the time and frequency domains for the normal stress component in the y or x direction at the epicenter and at a point on the surface of the plate on which the force is applied are presented. For comparison purposes results are also presented for a homogeneous plate whose properties are given by the static effective properties when the number of plies is large.

PROBLEM FORMULATION

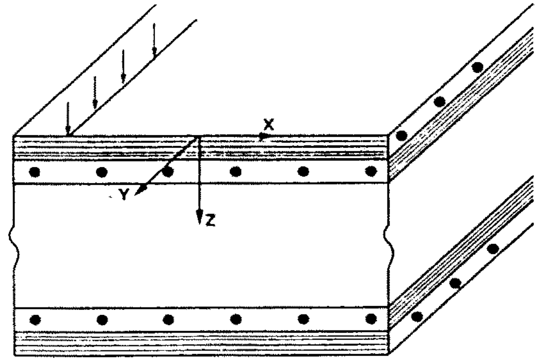
Consider a cross-ply laminated plate on one surface of which a line vertical force acts in the z-direction (Fig. 1). The line along which the force acts is taken to be parallel to either the x- or y-axis. The x-axis is chosen to be parallel to the fiber direction in the topmost ply. It will be assumed that the frequency content of the applied force is low enough so that each ply can be modeled as a transversely isotropic medium with the axis of symmetry parallel to the fibers. Thus for the problem under consideration the displacement **U** is independent of y for the configuration shown in Fig. 1. Also **U** has only nonzero components $u(x, z, t)$ and $w(x, z, t)$ in the x- and z-directions, respectively.

Now the relevant stress components in the ply with fibers parallel to the x-axis (0°) are related to the displacements as

$$\sigma_{xx} = C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial w}{\partial z}$$

$$\sigma_{zz} = C_{12}\frac{\partial u}{\partial x} + C_{33}\frac{\partial w}{\partial z}$$

$$\sigma_{zx} = C_{55}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right). \quad (1)$$



Laminated crossply plate with vertical line load

Fig. 1. Geometry of the laminated plate.

Here C_{ij} are the elastic stiffness coefficients.

In an earlier paper (Bouden and Datta, 1990) it was shown that the equations of motion in this play are satisfied if, following Buchwald (1961), it is assumed that

$$u = \frac{\partial \Theta}{\partial x}, \quad w = \frac{\partial \Phi}{\partial z} \tag{2}$$

provided Θ and Φ satisfy the equations:

$$(C_{12}+C_{55})\frac{\partial^4 \Phi}{\partial x^2 \partial z^2} + \left[C_{55}\frac{\partial^2}{\partial z^2} + C_{11}\frac{\partial^2}{\partial x^2} - \rho\frac{\partial^2}{\partial t^2}\right]\frac{\partial^2 \Theta}{\partial x^2} = 0 \tag{3}$$

$$(C_{12}+C_{55})\frac{\partial^4 \Theta}{\partial x^2 \partial z^2} + \left[C_{33}\frac{\partial^2}{\partial z^2} + C_{55}\frac{\partial^2}{\partial x^2} - \rho\frac{\partial^2}{\partial t^2}\right]\frac{\partial^2 \Phi}{\partial z^2} = 0. \tag{4}$$

Here ρ is the density. Note that for the 90° ply the governing equations are obtained by replacing C_{11} with C_{22} $(= C_{33})$ and C_{55} by C_{44}. Also, when the line of force is parallel to the x-axis then the governing equations for the 90° ply will be the same as above with the changes $u \to v$ and $x \to y$. Furthermore, the equations for the 0° ply in that case are then obtained by replacing C_{11} with C_{22} and C_{55} with C_{44}. In the following we focus our attention on eqns (1)–(3). Appropriate changes indicated above can then be made to get the solution valid for a different ply and for the line force along the x-axis.

The boundary conditions that are to be satisfied by the stresses and displacements are

$$\begin{aligned} &\sigma_{zz} = -\sigma_0(t)\delta(x-x_0), \quad \sigma_{zz} = 0, \quad z = 0 \\ &\sigma_{zz} = \sigma_{xz} = 0, \quad z = H. \end{aligned} \tag{5}$$

In addition, assuming that the plies are perfectly bonded, the continuity of U and σ_{zz} and σ_{zx} at the interfaces must be satisfied.

To solve eqns (3) and (4) we take the Fourier transforms with respect to x and t. Thus we write:

$$\Phi^*(k,z,\omega) = \int_{-\infty}^{\infty} \Phi(x,z,t)\,e^{-ikx+i\omega t}\,dx\,dt \tag{6}$$

$$\bar{\Theta}^*(k,z,\omega) = \int_{-\infty}^{\infty} \Theta(x,z,t)\,e^{-ikx+i\omega t}\,dx\,dt. \tag{7}$$

Then it is easily shown (Bouden and Datta, 1990) that using a local coordinate system for each ply with the origin at the top of the ply, the solutions for $\bar{\Theta}^*$ and $\bar{\Phi}^*$ can be written as

$$\begin{aligned} \bar{\Theta}^* &= F_1\Omega_1^+ + G_1\Omega_2^+ \\ \bar{\Phi}^* &= F_2\Omega_1^+ + G_2\Omega_2^+. \end{aligned} \tag{8}$$

The expressions for Ω can be written as

$$\Omega_1^+ = A_{11}e^{is_1z} + A_{12}e^{is_1(h-z)}$$

$$\Omega_2^+ = A_{21}e^{is_2z} + A_{22}e^{is_2(h-z)} \tag{9}$$

where

$$s_{1,2}^2 = \frac{-[\gamma k^2 - k_2^2(1+\beta)] \pm \sqrt{[\gamma k^2 - k_2^2(1+\beta)]^2 - 4\beta(k^2-k_2^2)(\alpha k^2 - k_2^2)}}{2\beta}. \tag{10}$$

The nondimensional material constants appearing in eqn (10) are $\alpha = C_{11}/C_{55}$, $\beta = C_{33}/C_{55}$, $\delta = 1 + C_{13}/C_{55}$, and $\gamma = 1 + \alpha\beta - \delta^2$. Also, we have taken h to be the thickness of a ply and $k_2^2 = \rho\omega^2/C_{55}$. The roots $s_{1,2}$ are chosen so that $\text{Im}\, s_{1,2} \geqslant 0$. The constants appearing in eqn (8) may be taken as $F_1 = G_1 = 1$ and

$$F_2 = \frac{k_2^2 - \alpha k^2 - s_1^2}{\delta s_1^2}, \quad G_2 = \frac{k_2^2 - \alpha k^2 - s_2^2}{\delta s_2^2}. \tag{11}$$

Using (8) in (1) and (2), the transforms of displacements and tractions can be written in the matrix form as follows:

$$\{\mathbf{u}\}_m = [E]_m[D]_m\{\mathbf{v}\}_m. \tag{12}$$

Here, m is the layer (ply) identifier,

$$\{\mathbf{u}\}_m = [u, w, \sigma_{zz}, \sigma_{zx}]_m^T \tag{13}$$

$$\{\mathbf{v}\}_m = [A_{11}, A_{21}, A_{12}, A_{22}]_m^T \tag{14}$$

and $[D]_m$ is a 4×4 diagonal matrix

$$\text{diag}\,[D]_m = [e^{is_1z}, e^{is_2z}, e^{is_1(h-z)}, e^{is_2(h-z)}]_m \tag{15}$$

and

$$[E]_m = \begin{bmatrix} ik & ik & ik & ik \\ is_1F_2 & is_2G_2 & -is_1F_2 & -is_2G_2 \\ C_{55}\Gamma_1 & C_{55}\Gamma_2 & C_{55}\Gamma_1 & C_{55}\Gamma_2 \\ -ks_1C_{55}(1+F_2) & -ks_2C_{55}(1+G_2) & ks_1C_{55}(1+F_2) & ks_2C_{55}(1+G_2) \end{bmatrix}_m. \tag{16}$$

Γ_1 and Γ_2 are given by

$$\Gamma_1 = (1-\delta)k^2 - \beta s_1^2F_2 \tag{17}$$

$$\Gamma_2 = (1-\delta)k^2 - \beta s_2^2G_2. \tag{18}$$

Equation (12) can be written in the partitioned form as

$$\{\mathbf{u}\}_m = \begin{Bmatrix} \mathbf{u}_D \\ \mathbf{u}_S \end{Bmatrix} = \begin{bmatrix} Q_{11}^m & Q_{12}^m \\ Q_{21}^m & Q_{22}^m \end{bmatrix} \begin{bmatrix} (D_{11}^m)_z & 0 \\ 0 & (D_{11}^m)_{\bar{z}} \end{bmatrix} \times \begin{Bmatrix} \mathbf{v}^- \\ \mathbf{v}^+ \end{Bmatrix}_m \tag{19}$$

where the subscript z stands for the depth level from the top of the ply at which $[D]$ is evaluated, and the subscripts D and S denote displacements and tractions, respectively.

Using the solution (9) for each ply and applying the boundary conditions enunciated before the following equations are obtained to solve for the $4n$ constants $\{v\}_m$ $(m = 1, \ldots, n)$ when there are n plies.

$$[Q^1_{21} \quad Q^1_{22}]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{is_1h} & 0 \\ 0 & 0 & 0 & e^{is_2h} \end{bmatrix}\begin{Bmatrix} v^- \\ v^+ \end{Bmatrix}_1 = \begin{Bmatrix} -\sigma_0^* e^{ikx_0} \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} Q^1_{11} & Q^1_{12} \\ Q^1_{21} & Q^1_{22} \end{bmatrix}\begin{bmatrix} e^{is_1h} & 0 & 0 & 0 \\ 0 & e^{is_2h} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_1 \begin{Bmatrix} v^- \\ v^+ \end{Bmatrix}_1 = \begin{bmatrix} Q^2_{11} & Q^2_{12} \\ Q^2_{21} & Q^2_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{is_1h} & 0 \\ 0 & 0 & 0 & e^{is_2h} \end{bmatrix}_2 \begin{Bmatrix} v^- \\ v^+ \end{Bmatrix}_2$$

$$[Q^n_{21} \quad Q^n_{22}]\begin{bmatrix} e^{is_1h} & 0 & 0 & 0 \\ 0 & e^{is_2h} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_n \begin{Bmatrix} v^- \\ v^+ \end{Bmatrix}_n = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \tag{20}$$

The $4n$ equations (20) can be solved for the unknown coefficients and then the transforms of the displacements and tractions at any point can be evaluated using eqn (12). Once the transforms are known then the time-domain is obtained by first evaluating the inverse Fourier transforms with respect to the spatial variable using an adaptive Clenshaw–Curtis scheme and then using an FFT. In the following numerical results are presented for a particular example.

NUMERICAL RESULTS AND DISCUSSION

Numerical results were obtained for a graphite fiber-reinforced epoxy cross-ply laminate. The properties of the 0° ply considered were the same as those in Datta *et al.* (1988), namely,

$$C_{11} = 160.73\ \text{GPa}, \quad C_{33} = 13.92\ \text{GPa}, \quad C_{13} = 6.44\ \text{GPa}, \quad C_{55} = 7.07\ \text{GPa},$$
$$C_{44} = 3.50\ \text{GPa}.$$

The ply lay-up considered was $[0^\circ/90^\circ]^n_s$ where n was taken to be 2, 4 and 8. The total thickness of the plate was 5.08 mm.

Figures 2 and 3 show the dispersion curves for propagation in the 0° and 90° directions, respectively. Also shown in these figures are the results when the layered plate is replaced by an equivalent homogeneous plate with properties obtained using the static effective medium approximation for a periodic layering (see Karunasena *et al.*, in press). For the particular ply properties considered these are:

$$\bar{C}_{11} = 87.32\ \text{GPa}, \quad \bar{C}_{13} = 6.68\ \text{GPa}, \quad \bar{C}_{33} = C_{33}, \quad \bar{C}_{44} = \bar{C}_{55} = 4.68\ \text{GPa}.$$

These figures show that the first flexural mode behaviors of the layered plate in the two directions are identical over the frequency range considered and are predicted well by the homogeneous plate model. It is also seen that the first longitudinal mode behaviors are somewhat different, but the differences are small. The behaviors of the higher modes are, however, vastly different from one another. The effects of increasing the number of laminae are shown in Figs 4 and 5. It is clear that as the number of layers increases the dispersions of higher modes are predicted more closely by the homogenized model. It may be concluded then that to predict the behavior of a laminated plate by a homogenized plate model there is a certain number of laminae that must be present. This number depends on the frequency content of the disturbance. In order to see this, results for the transient response of the plate are presented next.

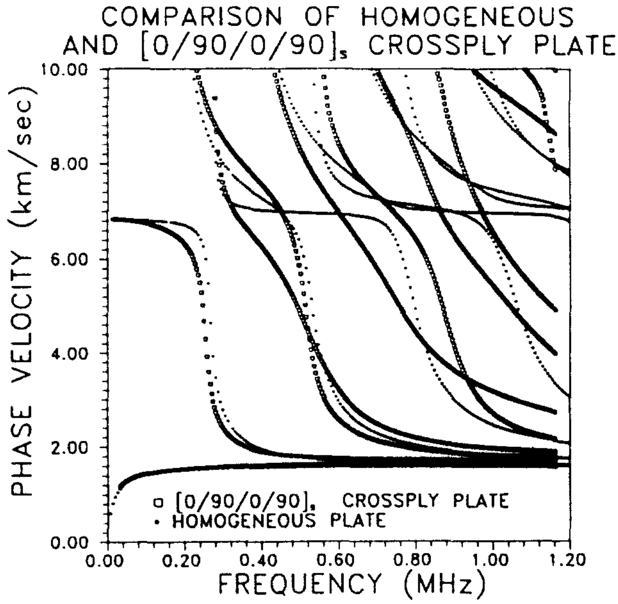


Fig. 2. Dispersion curves for the propagating modes in the 0° direction in an 8-ply ([0°/90°]$_s^2$) plate. Also shown are the results for the homogeneous plate.

Figures 6 and 7 present the frequency spectra of the vertical displacement component at a distance five times the thickness of the plate from the source. The time dependence of the source is taken as $\delta(t)$. Figure 6 shows the response when the line source is along the y-axis (at right angles to the fibers in the 0° ply), when it is along the x-axis (parallel to the fibers in the 0° ply), and when it is along the y-axis acting on a plate with homogeneous effective properties. It is seen that the spectra are qualitatively the same for the first two cases at low frequencies ($k_2H < 8$), but are quite different at high frequencies. Note that $k_2H = 8$ corresponds to a frequency of 0.5 MHz. As seen from Figs 2 and 3 the vertical response below this frequency is dominated by the flexural mode except near the cutoff frequency of S_1 mode. The response of the homogeneous plate and that in the first two cases show sharp peaks at cutoff frequencies of certain longitudinal modes: $S_1(P)$, $A_3(P)$, $S_4(P)$, and $A_5(P)$. The response of the homogeneous plate shows maxima also at the cutoff frequencies of certain flexural modes: $S_2(S)$, $S_3(S)$, $A_4(S)$, $S_5(S)$, and $A_6(S)$. There are other maxima

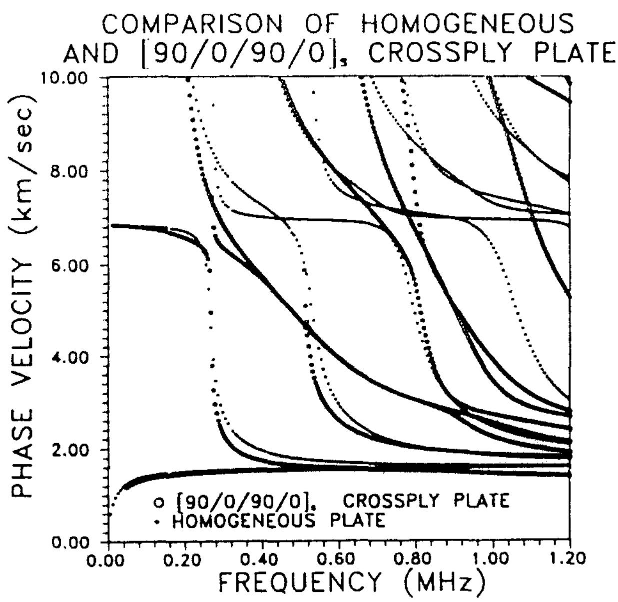


Fig. 3. Same as in Fig. 2 for propagation in the 90° direction.

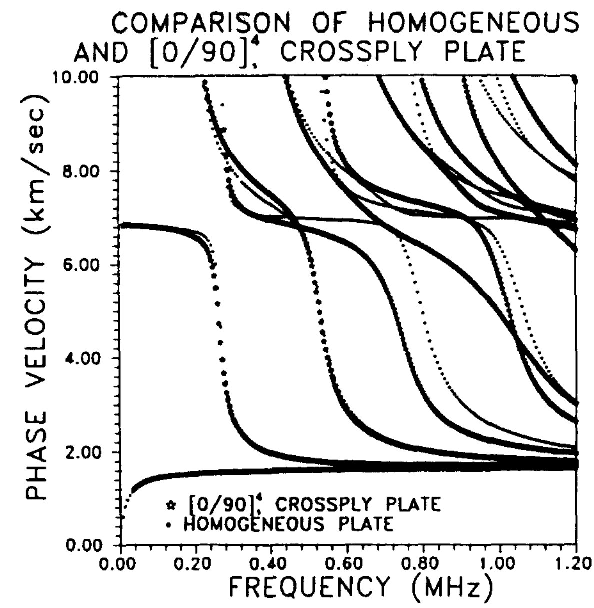


Fig. 4. Dispersion curves for propagation in the 0° direction in a 16-ply ([0°/90°]$_s^4$) plate.

in the response which do not seem to correspond to any cutoffs. Figure 7 shows the effect of increasing the number of plies to 16 in comparison with the results for the homogeneous and 8-ply plates. The results for the 16-ply plate and homogeneous plate are quite similar up to the cutoff frequency of $A_3(P)$ mode. This is consistent with the dispersion behavior shown in Fig. 4, where it is seen that the homogeneous model predictions agree closely with those of the layered model for frequencies less than about 0.6 MHz. Figures 8 and 9 show the transient response of the plate when the source is a Ricker pulse with center frequency, $f_c c_s/H$, at 0.21 MHz. The time dependence of this pulse is given by

$$\sigma_0(t) = (2\pi^2 f_c^2 \tau^2 - 1)\, e^{-\pi^2 f_c^2 \tau^2} \tag{21}$$

where $f_c = k_2 H/2\pi$, $\tau = t c_s/H$, and $c_s^2 = C_{55}/\rho$. It is seen from these figures that the results for the 16-ply and homogeneous plates are quite close.

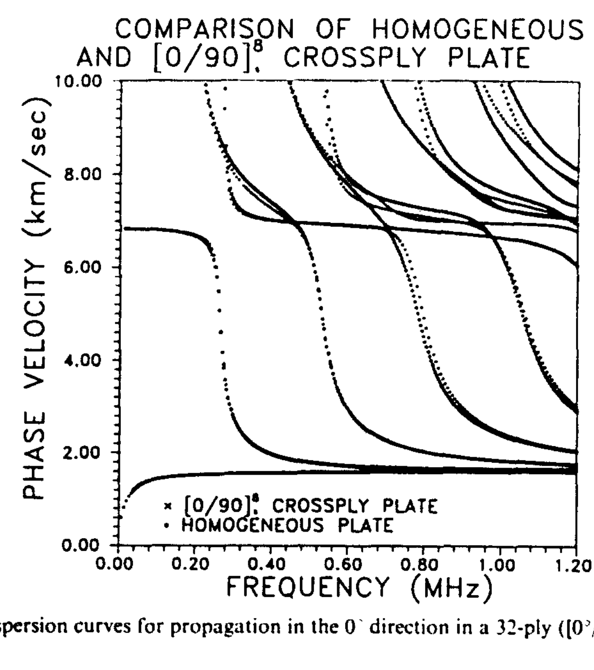


Fig. 5. Dispersion curves for propagation in the 0° direction in a 32-ply ([0°/90°]$_s^8$) plate.

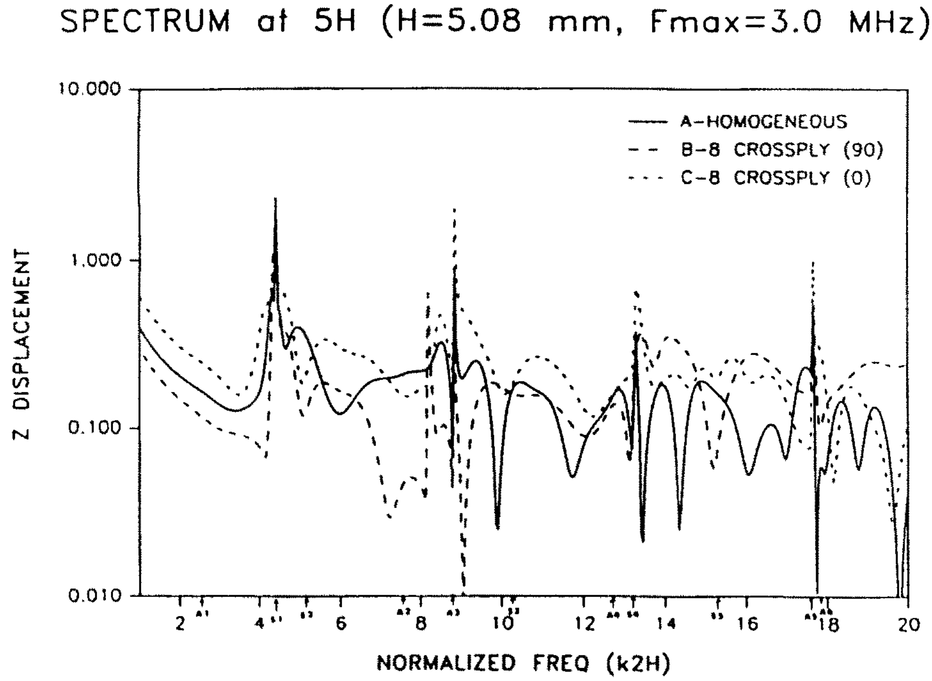


Fig. 6. Frequency spectrum of the z-displacement at a distance of $5H$ from the source where the source function is $\delta(t)$ in time. Comparison of the homogenized model and 8-ply plate predictions.

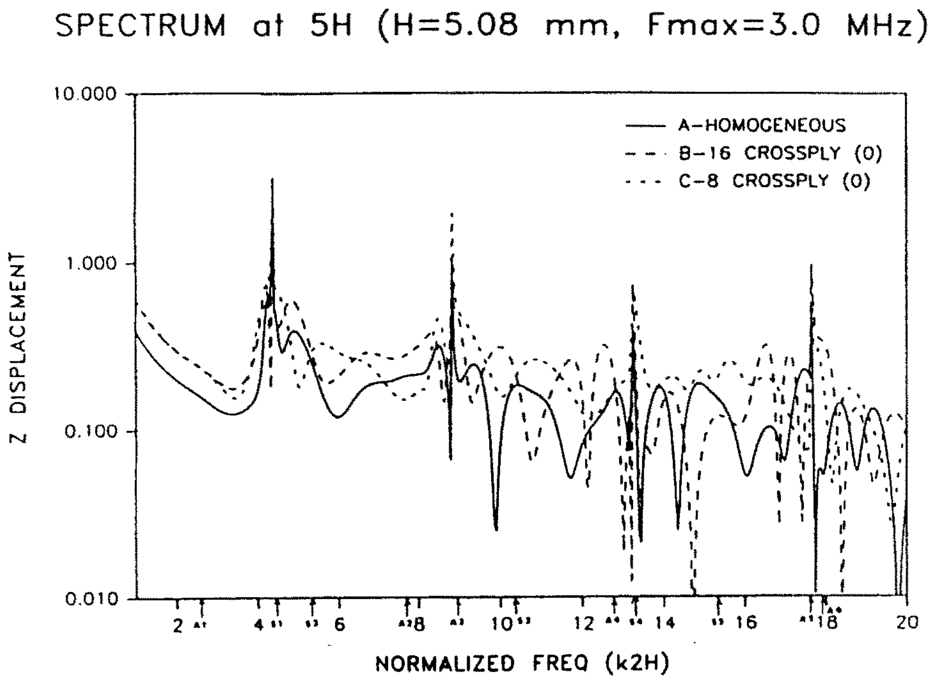


Fig. 7. Same as in Fig. 6. Comparison of the homogenized model, 8-ply and 16-ply plate predictions.

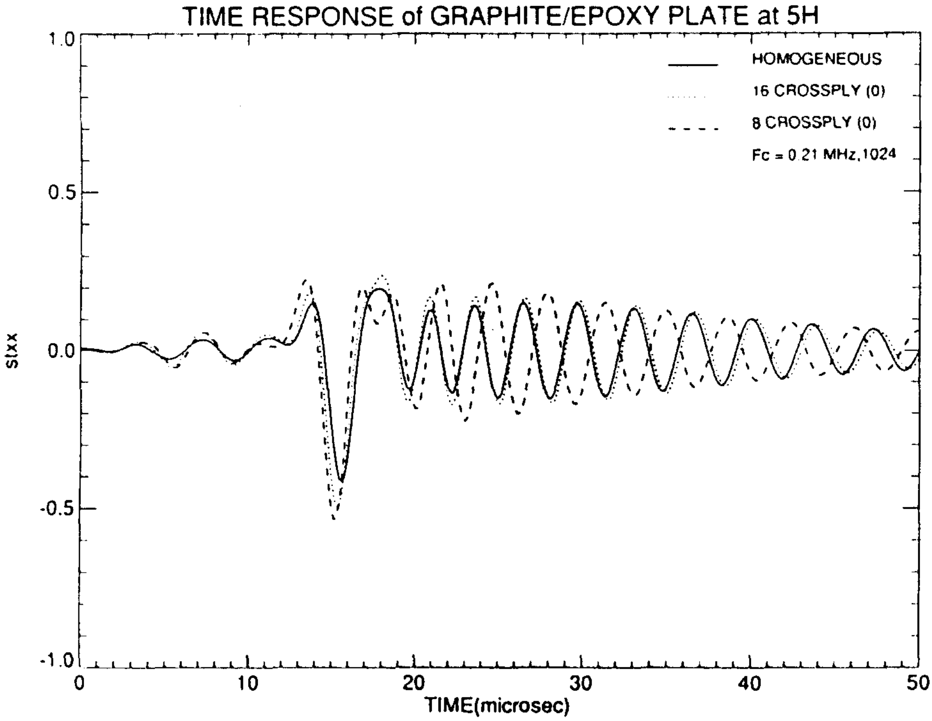


Fig. 8. The response in time of the horizontal strain at $5H$ distance from the source for a Ricker pulse with center frequency of 0.21 MHz. Comparison of the homogenized model and 8-ply plate predictions.

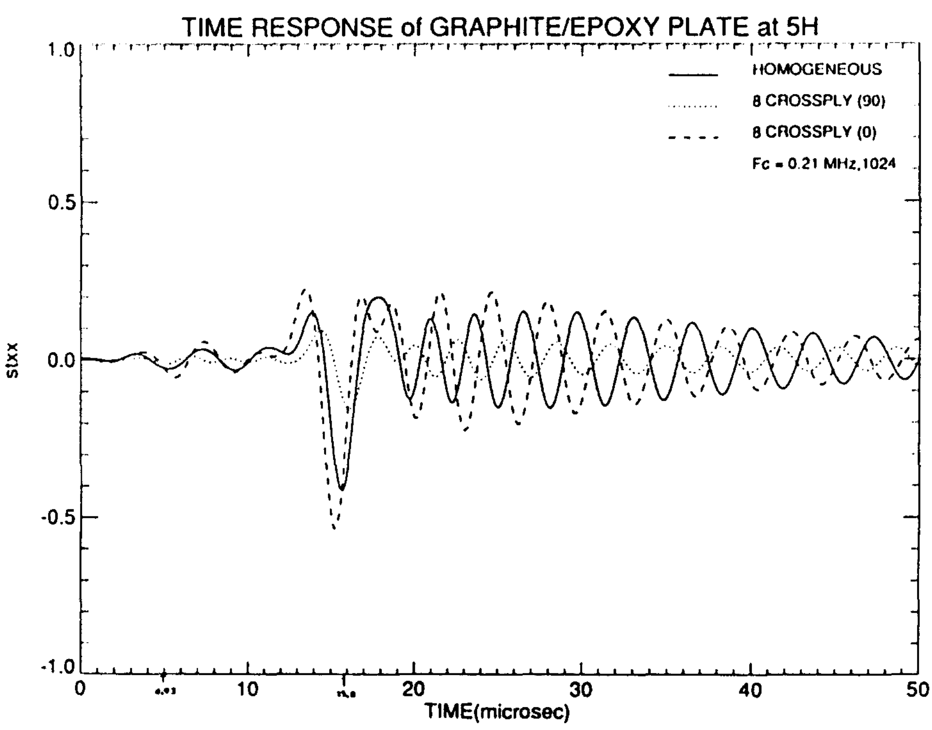


Fig. 9. Same as in Fig. 8. Comparison of the homogenized model, 8-ply and 16-ply plate predictions.

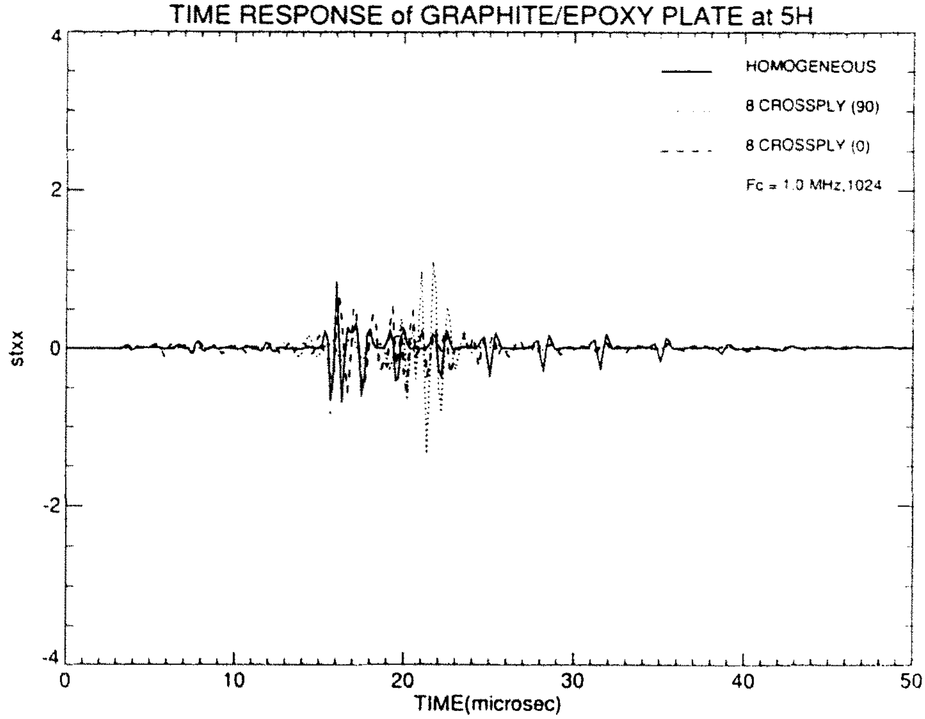


Fig. 10. Same as in Fig. 8 where the center frequency is 1.0 MHz.

Finally, Fig. 10 shows the time-response of the 8-ply plate when the center frequency is 1 MHz. The response of the plate for the two types of sources are found to be very different. This is consistent with large differences in the dispersion behavior as shown in Figs 2 and 3.

CONCLUSION

Model calculations have been presented showing the frequency and time responses in plane strain of a cross-ply plate with many plies due to a line source acting on the surface of the plate. Effect of increasing the number of plies on the response has been studied. It is found that the response of the multi-ply plate can be predicted closely by an effective homogeneous plate model within a certain frequency range if there is a certain minimum number of plies, this number depends on the frequency range of interest.

Acknowledgements—The work reported here was supported by a grant from the office of Naval Research, Solid Mechanics Program (Grant # N0014-91-J-1297; Scientific Officer—Dr. Y. D. S. Rajapakse). Partial support was received also from NASA (NAGW # 1388).

REFERENCES

Abubakar, I. (1962). Free vibrations of a transversely isotropic plate. *Q. J. Mech. Appl. Math.* **15**, 129–136.
Baylis, E. R. and Green, W. A. (1986). Flexural waves in fiber reinforced laminated plates. *J. Sound Vibr.* **110**, 1–26.
Bouden, M. and Datta, S. K. (1990). Rayleigh and Love waves in cladded anisotropic medium. *J. Appl. Mech.* **56**, 398–403.
Bratton, R. L., Datta, S. K. and Shah, A. H. (1989). Anisotropy effects on Lamb waves in composite plates. *Rev. Prog. in Quantitative Nondestructive Evaluation* (Edited by D. O. Thompson and D. E. Chimenti) **8A**, 197–204. Plenum Press, New York.
Bratton, R. L., Datta, S. K. and Shah, A. H. (1990). Propagation and scattering of Lamb waves in a fiber-reinforced composite plate. CSC Report 90-1, Center for Space Consruction, University of Colorado, Boulder.
Buchwald, V. T. (1961). Rayleigh waves in transversely isotropic media. *Q. J. Mech. Appl. Math.* **14**, 293–317.
Chang, C. and Sun, C. T. (1988). Acoustic emissions and transient elastic waves in an orthotropic laminate plate. *Composite Sci. Struct.* **33**, 213–236.
Chimenti, D. and Nayfeh, A. H. (1990). Ultrasonic reflection and guided wave propagation in biaxially laminated composite plates. *J. Acoust. Soc. Am.* **87**, 1409–1415.

Datta, S. K., Shah, A. H., Bratton, R. L. and Chakraborty, T. (1988). Wave propagation in laminated composite plates. *J. Acoust. Soc. Am.* **83**, 2020–2026.

Datta, S. K., Achenbach, J. D. and Rajapakse, Y. S. (Eds) (1990). Elastic waves and ultrasonic nondestructive evaluation. North-Holland, Amsterdam.

Dayal, V. and Kinra, V. K. (1989). Leaky Lamb waves in an anisotropic plate. I: An exact solution and experiments. *J. Acoust. Soc. Am.* **85**, 2268–2276.

Dong, S. B. and Hwang, K. H. (1985). Edge vibrations in laminated composite plates. *J. Appl. Mech.* **52**, 433–438.

Dong, S. B. and Nelson, R. B. (1972). On natural vibrations and waves in laminated orthotropic plates. *J. Appl. Mech.* **39**, 739–745.

Karunasena, W. M., Bratton, R. L., Datta, S. K. and Shah, A. H. (In press). Elastic wave propagation in laminated composite plates. *J. Engng Mater. Tech.*

Karunasena, W., Shah, A. H. and Datta, S. K. (In press). Wave propagation is a multi-layered laminated cross-ply composite plate. *J. Appl. Mech.*

Kaul, R. K. and Mindlin, R. D. (1962). Frequency spectrum of a monoclinic crystal plate. *J. Acoust. Soc. Am.* **34**, 1902–1910.

Li, Y. and Thompson, R. B. (1990). Influence of anisotropy on the dispersion characteristics of guided ultrasonic plate models. *J. Acoust. Soc. Am.* **87**, 1911–1931.

Mal, A. K. (1988). Wave propagation in layered composite laminates under periodic surface loads. *Wave Motion.* **10**, 257–266.

Mal, A. K. and Bar-Cohen, Y. (1989). Ultrasonic NDE of thick composite laminates. *Rev. Prog. in Quantitative Nondestructive Evaluation* (Edited by D. O. Thompson and D. E. Chimenti) **8B**, 1551–1558. Plenum Press, New York.

Mal, A. K. and Ting, T. C. (1988). *Wave Propagation in Structural Composites.* The American Society of Mechanical Engineers, New York.

Mal, A. K., Yin, C.-C. and Bar-Cohen, Y. (In press). Analysis of acoustic pulses reflected from fiber-reinforced composite laminates. *J. Appl. Mech.*

Moon, F. C. (1972). Wave surface due to impact on anisotropic plates. *J. Composite Mater.* **6**, 62–79.

Nayfeh, A. H. and Chimenti, D. (1988a). Ultrasonic wave reflection from liquid-coupled orthotropic plates with applications to fibrous composites. *J. Appl. Mech.* **55**, 863–870.

Nayfeh, A. H. and Chimenti, D. (1988b). Propagation of guided waves in fluid-coupled plates of fiber-reinforced composite. *J. Acoust. Soc. Am.* **83**, 1736–1743.

Postma, G. W. (1955). Wave propagation in a stratified medium. *Geophysics* **20**, 780–806.

Press, W. H., Flannery, B. P., Teukolsky, S. A. and Vetterling, W. T. (1988). *Numerical Recipes: The Art of Scientific Computing.* Cambridge University Press, Cambridge.

Solie, L. P. and Auld, B. A. (1973). Elastic waves in anisotropic plates. *J. Acoust. Soc. Am.* **54**, 50–65.

Whitney, J. M. and Sun, C. T. (1973). A higher order theory for extensional motion of laminated composites. *J. Sound Vibr.* **30**, 85–97.